Integrated Numerical Simulation in Advanced Geometric Designing Systems

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Abstract:

Actual engineering processes can be simulated and analyzed in virtual environments by using Computer Aided Design (CAD) methods. The CAD is presented by geometrical parameter as Non-Uniform Rational B- Splines (NURBS), B-spline and T-splines. Stress and strain of produced parts can also be simulated in virtual environments by using the CAD models in order to be analyzed and controlled. The Finite Element Method (FEM) is a very common analysis method in order to analyze CAD models. Efficiency of FEM is under influence of many parameters such as approximation, polynomial based geometry, mesh generation and mesh refinement process, sliding contact and flows about aerodynamic shapes. The isogeometric analysis is introduced to eliminate the analysis costs related to geometry clean-up, defeaturing and mesh generation. The integrated numerical simulation in the part designing is development and implementation of the next generation design and simulation methods based on isogeometric analysis. Polynomial splines over hierarchical T-meshes (PHT-splines) will be employed to construct exact geometric models. PHT-splines will be extended to RHT- spline (Rational spline over Hierarchical T-meshes) as rational spline in order to capture a wider class of geometries. So, a general framework of unifying preprocessing as well as designing using isogemetric analysis can be provided in order to improve efficiency of produced parts.

Keywords: Finite Element Method (FEM), Isogeometric analysis, NURBS Modeling

1- Introduction

The Computational methods are based on the geometry of simulated parts in virtual environments by presenting diverse spatial scales of real feathers. Also, topology is a branch of mathematics to present material properties of objects under continuous deformations such as stretching and bending. Space dimension as well as transformation are used by topology science in terms of developing concepts through geometry and basic theory.

Designers and engineers can present their new ideas using Computer Aided Design (CAD) which is start point of converting their perception to new products. It presents computer models of objects by using geometrical parameters in virtual environments. Three dimensional (3D) representations as well as wireframe views with ability of changing relevant parameters are some advantages of simulated models of objects in virtual environments.

Today fundamental of most CAD systems are based on the spline basis function as well as Non-Uniform Rational B-Splines (NURBS). NURBS is a mathematical model of free form surfaces and shapes which is used commonly in CAD systems by offering great flexibility and adaptability. In general, NURBS is generalized and improved upon the traditional piecewise polynomial basis function. The traditional piecewise polynomial basis functions are improved

by NURBS modeling in order to provides flexibility, unprecedented accuracy and editing predictability. A set of weighted control points and a knot vector are main parts of a NURBS curve. B-spline is a spline function for modeling of free form surfaces with ability of smoothness, domain partition and minimal support to a given degree.
B-spline curve frequently refers to a spline curve which is parameterized by spline functions in order to present linear combinations of B-splines. Thus, B-spline curves can present many important properties by Bézier curves.
Moreover, T-splines can describe arbitrary topological shapes with preserving suitable basis of a smooth analysis and high level of variety and complexity. So, the property can improve efficiency of the finite elements analysis of many engineering problems. It can express complex surfaces with high levels of accuracy in details. Designer can add control points to every section of needed surfaces. As a result, T- Splines surface will have up to 70% fewer control points in comparison with NURBS surfaces. T-junction of T-Splines is a main difference between T-Splines and NURBS surfaces. It allows lines of detail to end elegantly in order to provide smooth surfaces at T-points. Modeled surfaces by T-Spline can describe partial isoparms using T-points which is a vertex of an isoparmonly on the one side of surfaces. CAD systems can create extrusions, holes and other unique features by T-Spline surface easily. T-Splines can describe such features in a single surface using a special point as star point which allows a single T- spline surface to be non-rectangular. But, NURBS require multiple surfaces for merging or a poly surface for such objects which makes second difference between T-Splines and NURBS. T-points and star points of T-Splines surfaces provide many advantages such as abilities of editing surfaces easily and grabbing a point anywhere on the surface. As a result, the modeled surface will stay smooth and free form by eliminating gaps between simulated feathers.
Polynomial Splines over hierarchical T-meshes (PHT-splines) is a new introduced type of splines and generalized B-splines over hierarchical T-meshes. It is presented to model geometric objects more efficient. The new splines can efficiently describe objects in order to fit open or closed mesh models. In the description, only linear systems of equations with a few unknowns are involved.
The fundamental functions of PHT-splines have the same important properties of B-splines such as non- negativity, local support and partition of unity [1]. Thus PHT-splines are a generalization of B-splines over hierarchical T-meshes. Given a rectangular domain a T-mesh is a partition of the domain and it is basically a rectangular grid that allows T-junctions [1]. It is assumed that the end points of each grid line in the T-mesh must be on two other grid lines and each cell or facet in the grid must be a rectangle [1]. Fig. 1 shows an example of a T-mesh [1].

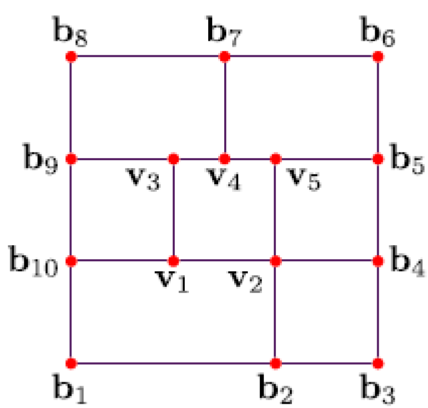

Fig. 1. An example of T-mesh.

A hierarchical T-mesh is a special type of T-mesh which has a natural level structure. One generally starts from a TP mesh (level 0). From level k to level k+1, one subdivides a cell at level k into four sub cells which are cells at level k +1 [1]. For simplicity, each cell has been subdivided by connecting the middle points of the opposite edges with two straight lines [1]. Fig. 2 illustrates the process of generating a hierarchical T- mesh [1].

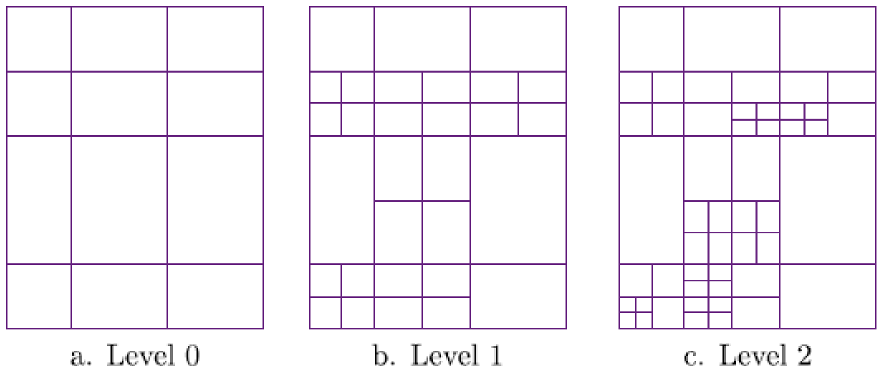

Fig. 2. A hierarchical T-mesh.

Given a T-mesh **Γ**, let **F** denote all the cells in **Γ** and **Ω** the region occupied by all the cells in **Γ**,

S (m, n,α, β, Γ): = { $s(x,y) \in C^{\alpha,\beta}(\Omega)|_{\phi} \in p_{mn}$ for any $\phi \in$ **F** }**,** (1)

Where p_{mn} is the space of all the polynomials with bi-degree (m, n), and $C^{\alpha,\beta}(\Omega)$ is the space consisting of all the bivariate functions which are continuous in Ω, with order α along the x-direction and with order β along the y-direction. It is obvious that S (m, n,α, β, Γ) is a linear space, which is called the spline space over the given T-mesh Γ. For any function b(u, v), its function value b(u, v), two partial derivatives of first order and mixed partial derivative are

$$b_u(u,v) = \frac{\partial}{\partial u} b\,(u,v)|_{(u,v)},$$
$$b_v(u,v) = \frac{\partial}{\partial v} b\,(u,v)|_{(u,v)}, \qquad (2)$$
$$b_{uv}(u,v) = \frac{\partial^2}{\partial u \partial v} b\,(u,v)|_{(u,v)},$$

At some point (u_0, v_0) are called the geometric information of $b\,(u,v)$ at point(u_0, v_0). Given a hierarchical T-mesh Γ, suppose the basis functions are $\{b_j^k(u,v)\}$, j = 1, . . . , N, k = 0, . . . , 3. Here N is the number of basis vertices. Then a spline surface over Γ can be defined as,

$$s(u,v) = \sum_{j=1}^{N} \sum_{k=0}^{3} C_j^k b_j^k(u,v), \qquad (3)$$

Where C_j^k are the control points associated with the jth basis vertex.

Efficiency of simulation methods is evaluated by using testing techniques to determine dynamic relations between a system of parts. So, different levels of numerical solid and fluid functions are considered during and after the design stages.

Finite Element Method (FEM) is very common test where the geometry is represented by piecewise low order polynomials. It is a numerical, mathematical and analytical technique test method to present approximate solutions of partial differential equations (PDE) as well as integral equations of physical problems. Technique of solution approach is based on the method of eliminating completely the differential equation (steady state problems) or rendering the PDE into an approximating system of ordinary differential equations. The solution will be integrated numerically using standard techniques such as Euler's method, Runge-Kutta, etc. A wide range of engineering as well as physical problems can be considered to be analyzed by using the FEM. The method presents an efficient tool to solve and find answer of partial differential equations of engineering problems with complicated domains. FEM technique divides surfaces to small elements by mesh generating methods. A sample surface which to be meshed for FEM analysis is shown in Fig. 3 (a). A possible mesh using triangular elements is shown in Fig. 3 (b). Rectangular elements and Rectangular and quadrilateral elements are shown in Fig. 3 (d) and Fig. 3 (e) respectively. In general, the ratio of the largest characteristic dimension of an element to the smallest characteristic dimension is known as the aspect ratio [2]. Large aspect ratios increase the inaccuracy of the finite element representation and have a detrimental effect on convergence of finite element solutions [3]. An aspect ratio of 1 is ideal but cannot always be maintained. Finite element software packages provide warnings when an element's aspect ratio exceeds some predetermined limit. As a result, the remesh process or changing meshing model is necessary [3].

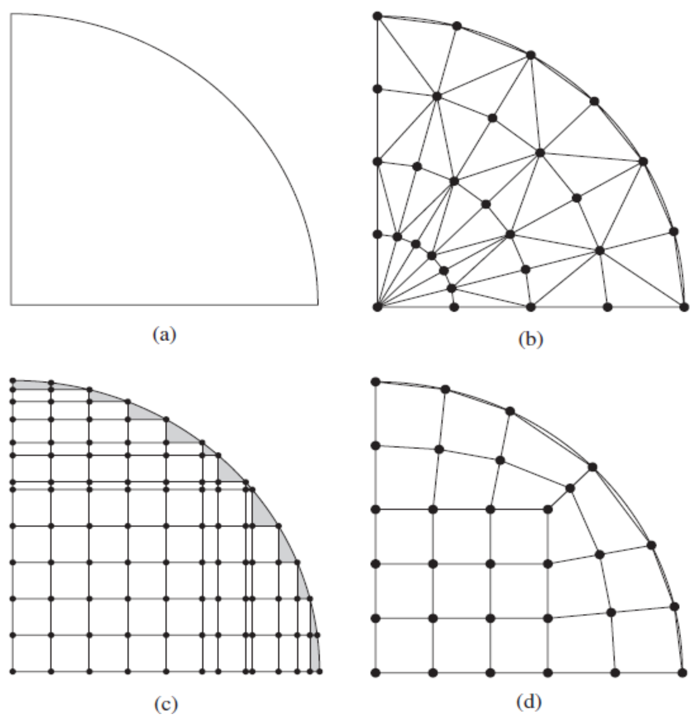

Fig. 3. A domain to be modeled (a) and different models of mesh generation (b), (c) and (d) [3].

In many cases, 80% or more control points of NURBS models are superfluous. By contrast, T- spline models typically require only 20% of the control points in comparison with to NURBS models. Furthermore, T- spline control mesh is allowed to have partial rows of control points by their T-junctions. For a designer, fewer control points mean lesser amount of computational time by reducing analysis volume. Refinement process which is adding new control points to a control mesh without changing the surface in terms of mesh revision is an important basic operation used by designer. Refinement process in the NURBS modeling operations can create limitation as well as time consuming applications.

T-junctions enable T-spline to locally refined by editing partial rows of control points. Most objects need several NURBS surfaces to be modeled as a result of having a rectangular topology for a single NURBS surface. It is also another limitation of NURBs modeling processes. Moreover, it is difficult to merge multiple NURBS surfaces within a single, smooth, free form and watertight model especially if corners of valence other than four are introduced. T-junctions of T-spline surface make it possible to merge together several NURBS surfaces in to a PHT-spline. Furthermore, it is mathematically impossible for a trimmed NURBS to accurately present the intersection of two

NURBS surfaces without introducing any gaps in the designed model. It is another serious inherited problem in NURBS surfaces. The problem is a major cause of the incompatibility between CAD and analysis software [4].

A NURBS surface can simplifies efficiently by converting to a PHT-spline which dramatically reduces the superfluous control points of the NURBS surface. Furthermore, PHT-splines provide several important types of geometry processing with natural and efficient manner. PHT-spline can present assembly of tensor-product spline with simple shapes using coarser T-meshes. PHT-splines not only inherit the main properties of T-splines such as adaptively but also exhibit more advantages over T-splines. PHT-splines are polynomial instead of rational structure of T-splines. So, the refinement algorithm of PHT-splines is local and simple. The conversion between NURBS and PHT-splines is very fast and simple, while conversion between NURBS and T-splines is a bottleneck of T-splines in practical applications. In comparison with T- splines and hierarchical B-splines, PHT splines have a set of basic functions. It is a necessity in some theoretical analysis and applications, while hierarchical B-splines have a redundant set of 'basis functions' [1]. On the other hand, hierarchical B-splines require a very special hierarchical T mesh structure due to their refinement scheme, while PHT-splines work over arbitrary hierarchical T-meshes [1].

Difficulties of CAD simulation systems create many problems for mesh generating process of geometry analysis using FEM. Many difficulties encountered with FEM emanate from its approximate, polynomial based geometry such as mesh generation, mesh refinement, sliding contact, flows about aerodynamic shapes, buckling of thin shells, etc. [5]. It is estimated that about 80% of overall analysis time is devoted to mesh generation in the automotive, aerospace, and ship building industries [4]. Presenting more powerful descriptions of geometry characters is demand of geometry analysis by computational techniques as FEA. Converting data between CAD and FEA is necessary process for many analysis studies and it is complicated process when there is different computational geometric approach for each segment. As a result, a methodology must be presented for bridging the gap between CAD and finite element analysis (FEA). The isogeometric paradigm can be invoked for eliminating problems of CAD surfaces analysis, associated with geometry clean-up, defeaturing and mesh generation which is proposed by Hughes et al. [4].

Isogeomtric analysis is computational approach of finite element analysis (FEA) into conventional NURBS, B-spline, T spline and PHT-splines in order to offer integrated analysis technique.

Common spline basis of geometric modeling systems and finite element analysis of a given object are used in the Isogeomtric analysis to fill the gap of CAD and

FEM. It employs directly complex CAD geometries in the FEA application for testing and adjusting models using a common data set. Eliminating and reducing the approximation of the computational domain as well as erasing demand of remeshing are some preferences of the method. Main advantage of the proposed generalized isogeometric analysis is wide range of applications such as analysis of models based on CAD designs, surface models and point clouds generated by laser scanners. Diagram of Isogeomtric Analysis is shown in the Fig. 4.

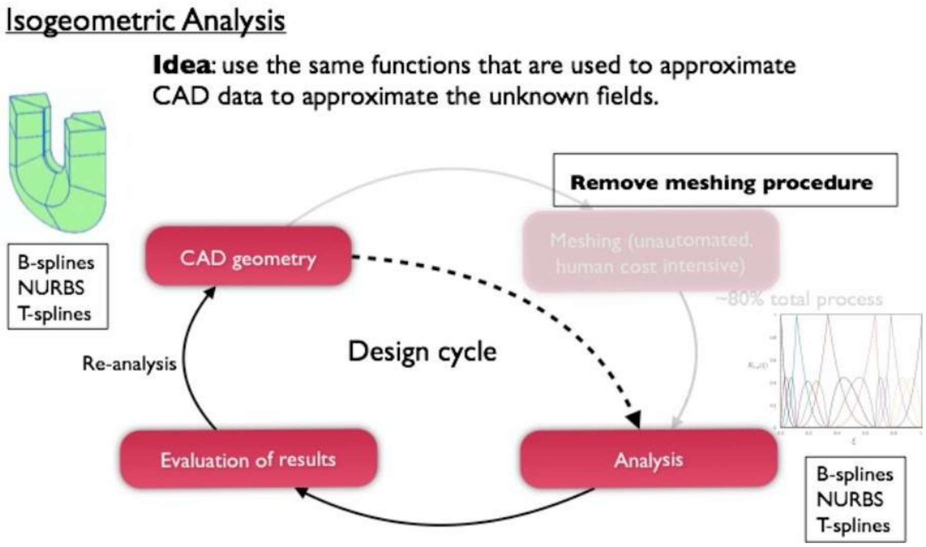

Fig. 4. Diagram of Isogeomtric Analysis.

Finite element analysis can model the topology of a domain in an accurate fashion but the geometry of simulated features are approximate [5]. CAD accurately represents the geometry while the topology has historically incorrect [5]. Isogeomtric analysis attempts to model both topology and geometry accuracy [5]. Fig. 5 shows comparison of modeling abilities in topology and geometry by FEM, CAD and IGA.

	Topology	Geometry
Finite Element Analysis	√	×
Computer Aided Design(CAD)	×	√
Isogeometric Analysis	√	√

Fig. 5. Comparison of Topology and Geometry Modeling by FEM, CAD and IGA.

Isogeometric analysis can use both of NURBS and T-splines surfaces which provide advantageous in comparison with traditional finite element analysis. It includes not only in the convergence of the analysis answers but also in accuracy of results. In the NURBS surfaces, tensor-product structure makes local refinement impossible. It depends to some variant as the structure of the mesh. Therefore, the refinement process is not local. But, T-splines have the ability of local refinement. For providing a same rules, several theoretically computation methods such as the linear independence of the blending functions are still open and under research. Polynomial splines over hierarchical T-meshes (PHT-splines) are generalized B-splines over hierarchical T-meshes with ability of local refinement by a simple algorithm. PHT-splines have great flexibility and adaptability for geometrical modeling [1]. Used Geometry in CAD and FEM has different descriptions. CAD and isogeometric analysis use the same geometry description. The process and refinement steps of FEM and IGA are shown in Fig. 6.

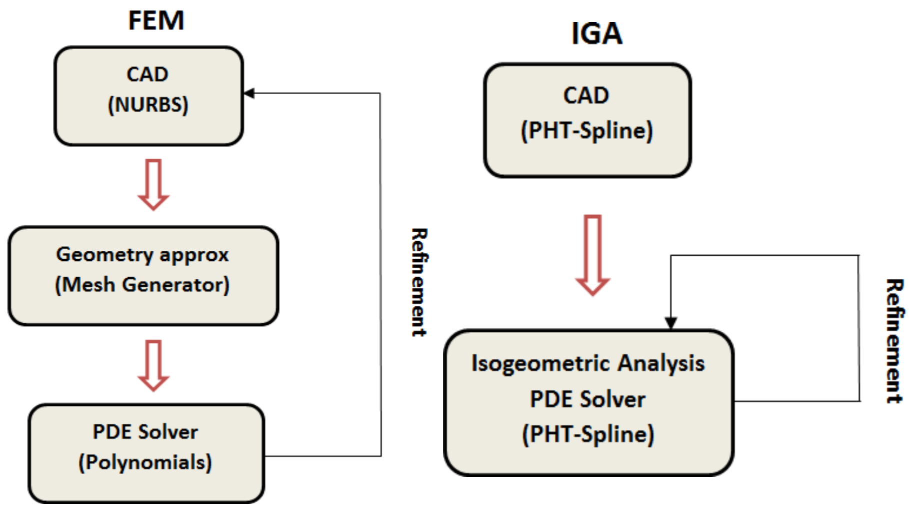

Fig. 6. Differences between FEM and IGM in process and refinement.

The CAD systems characters are based on geometry aspects of models. But in the CAE, three-dimensional of geometry representations are necessary. So, the process can be designed and presented to generate volume models in CAE from CAD surface models in order to connect the modeling systems. On the other hand, geometric models of CAD systems have to be re-approximated by CAE-software. The processes are complex and time consuming which are not desired in CAE analysis. Production of volume model becomes more intricate when most CAD functions are based on a tensor-product form with hexahedra – type meshes. But, exact geometry of CAD models without any effect of model discretization is available using isogeomtric analysis. Fig. 7 shows process of physical problem identification to the output of the system in modeling adaptively by isogeometric analysis.

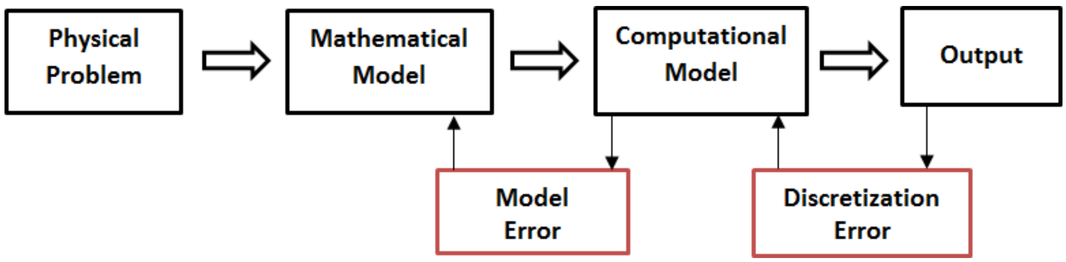

Fig. 7. Mathematical Model, Computational Model, Error Estimation and Model Adaptively

After determination of physical problems, mathematical model should be chosen according to a physical problem such as thin-shell model or a three-dimensional model. Physical problem are discretized by isogeometric shape functions. Types of errors are as the discretization error and the model error. Most common error estimators are as

- **Residual based error estimators** are used and developed in isogeometric analysis studies [6].
- **Recovery based error estimators** is developed based on using the computed solution in order to produce a smoother approximation of part by sampling the values (or its derivatives) at certain points in the domain and by using a weighted average to produce a more accurate approximation to u (or its gradient) at the nodes, the process can be accomplished. In order to produce the recovered solution, these averaged values can be fitted by an interpolation algorithm.
- **Goal oriented error estimators** are extended and developed by researchers when they found it is useful to estimate the error in some quantity of interest and they called it Goals algorithms. Oden et al. [7,8] presented and developed the idea of goal-oriented error estimators especially for multi scale error modeling. In order to provide multi scale modeling, a predefined base-model was enhanced until a minimum error was reached in a certain quantity of interest. As is shown in the Fig.8, handling discretization errors is by using goal-oriented error estimators which needed adaptively refine (or coarsen) a given discretization while goals algorithm focus on the model error.

Optimal and reduced quadrature rules for tensor product and hierarchically refined splines in isogeometric analysis is presented by Hiemstra et al. [9]. Also, Nguyen-Thanh et al. [10] presented isogeometric analysis of large-deformation thin shells using RHT-splines for multiple-patch coupling. Jia et al. [11] presented reproducing kernel triangular B-spline-based FEM for solving PDEs. Moreover, an isogeometric boundary element approach using T-splines for Shape optimization directly from CAD models is presented by Lian et al. [12]. Valizadeh et al. [13] presented NURBS-based finite element analysis of functionally graded plates: static bending, vibration, buckling and flutter. Also, a refined quasi-3D isogeometric analysis for functionally graded microplates based on the modified couple stress theory is presented by Nguyen et al. [14]. A two-dimensional isogeometric boundary element method for elastostatic analysis is also presented by Simpson et al. [15].

The Homotopy Perturbation Method is used by Nourazar et al. [16] to obtain the exact solution of Newell-Whitehead-Segel Equation. To obtain the exact solution of the Burgers-Huxley as well as Fitzhugh–Nagumo non-linear differential equations, application of the Homotopy Perturbation Method is investigated by Nourazar et al. [17,18] .The Variational Iteration Method and Homotopy Perturbation Method are used by Soori and Nourazar [19] in order to obtain the exact solution of nonlinear differential equations. To obtain the exact solution of nonlinear differential equation The variational iteration method and the homotopy perturbation method to the exact solution of the Fisher Type Equation is presented by Soori et al. [20]. The Variational Iteration Method and the Homotopy Perturbation Method to the Exact Solution of the generalized Burgers-Fisher Equation is used by Soori [21] in order to obtain the exact solution of nonlinear differential equation. To present the capabilities of the semi analytical methods in obtaining the exact solution of nonlinear differential equation, a Comparison between the Variational Iteration Method and the Homotopy Perturbation Method for the Burgers-Huxley Equation is presented by Soori [22]. The variational iteration method is used by Soori et al. [23] to obtain the exact solution of the Newell-Whitehead-Segel Equation. Also, the variational iteration method is used by Soori [24,25] to Solve the Korteweg-de Vries-Burgers Equation and Fitzhugh–Nagumo non-linear differential equations. To obtain the series solution of the Weakly-Singular Kernel Volterra Integro-Differential Equations, the Combined Laplace-Adomian Method is used by Soori [26].

The proposed research project is about the development and implementation of the next generation design and simulation methods based on isogeometric analysis. In order to present more advantages in comparison with NURBS surfaces, a new type of splines-polynomial splines over hierarchical T-meshes (PHT-splines) is employed to construct exact geometric models. We will extent the PHT-splines to a RHT- spline (Rational spline over Hierarchical T-meshes) as rational spline which can capture and demonstrate a wider class of geometries. The recovery based error estimators and Goals-oriented error estimators will be extended to model thin structures in terms of using the isogeometric analysis. In the first step, appropriate error estimators in the context of isogeometric analysis will be developed. Next, it will be extended to goals algorithms. Designed feathers as CAD parts which simulated by PHT-splines will be also evaluated and improved. It will be extended to RHT-spline in proposing accurate design and simulation methods by using isogeromteric analysis. This objective will be achieved by developing improved numerical methods for generalized isogeometric finite elements and robust adaptive refinement algorithms for unstructured and Cartesian meshes. It can also be exploited into isogeometric analysis which is a key link between CAD and CAE. Using isogemteric analysis, both topology and geometry of destined feathers will be tested and problems of FEM analysis such as mesh generation, mesh refinement with long process and only topology analysis will be illuminated. Refinements process is also easily implemented and exact geometry is maintained at all levels without the need of subsequent communication with a CAD description. For purposes of analysis, the basis is refined by feedback information and its order elevated without changing the geometry or its parameterization. In analysis of CAD systems, the feathers which are modeled by PHT- splines will be considered. This selection is as a result of flexibility and adaptability of geometrical modeling by PHT-splines. Furthermore, refinement algorithm is local and simple due to the nature of hierarchical T-meshes. Therefore, the analysis processes become more accurate and reliable.

2- Research methodology

The method is gathering all necessary information about demands of the project such as articles, books and internet pages. Then, the information and data will be used in computer aided design and analysis system such as CAD and FEM software with regard to the isogeometric analysis. Every gap will be also filled by programming languages such as C++, MATLAB, FORTRAN and Visual Basic. The results will be tested and evaluated by industrial cooperation in order to estimate accuracy and reliability of the analysis method. Objectives and targets of the project will be reviewed in terms of correcting and completing pervious phases by getting feedback. As a result, the research will be reached to a methodology and products according to the project schedule.

3- Conclusion

The aim of the study is providing and presenting a general framework of unifying pre-processing and designing with numerical and analytical technique as isogeometric analysis. The framework will be applied into the most common and popular methods employed in pre-processing design and analysis of CAD designed feathers using PHT-splines method. The key outcome of the research project can be presented as a system and methodology which can improve efficacy and reliability of parts designing and simulating. The aim of the developed system is designing and developing a state of art programming tool using the numerical schemes and isogeometric analysis at the INSIST project. Extension of isogeometric analysis for presenting unification of CAD, CAE and analysis techniques are aims of the project to unify pre-processing and analysis system. So, a productive method will be presented in the project in order to improve efficiency of designing and testing parts production.

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